

# Relating High School Mathematics through Sound and Images

*Gail Rosen<sup>1</sup>, Marion Usselman<sup>2</sup>, and Donna Llewellyn<sup>3</sup>*

**Abstract** – In this project we developed a series of computer laboratories using Mathwork’s Matlab<sup>®</sup> software to help students relate to math concepts through a “hands-on” multimedia approach. Initially, the Matlab<sup>®</sup> laboratories exposed students to computer programming and debugging, plotting graphs, and solving equations. The labs then extended to exploring mathematical concepts through audio and visual applications. For example one lab related a rotation matrix learned in trigonometry to animating an object on the computer. Another explored parameters of a sine wave and how they related to music and telephone touch-tones. The challenge of designing the labs was to facilitate critical thinking and exploration while minimizing the drudgery of debugging. In this paper, we will present three of the labs and discuss how they relate to the trigonometry/pre-calculus curriculum, the response of the high school students, and the lessons learned by the graduate student instructor.

*Keywords: Audio laboratories, Image laboratories, Trigonometry, Pre-Calculus*

## INTRODUCTION

*“In highly-developed countries, many children are exposed to advanced technology at a young age through computers, video games, digital audio and video, and electronic mail. Because of modern-day pre-college curricula, however, many of these children will never learn about the mathematical and scientific underpinnings of our society’s ubiquitous high technology. Such a situation is likely to undermine progress in technology unless society is able to foster a significantly-larger group of prospective engineering students.” [1]*

IEEE and engineers across the country recognize that in order to develop a sufficiently large population of new engineers, it is essential that we connect everyday technology to mathematical and scientific concepts. Rice University’s INFINITY project has been a pioneering collaboration between educators, administrators, and industry leaders to establish an exciting engineering curriculum at the high school level that can motivate and attract students [1, 2]. The curriculum is designed specifically to appeal to the intended audience—through the universal medium of music. “We as authors recognized the importance of music to motivate the understanding of fundamental concepts in one-dimensional signals and their manipulation. Almost everybody listens to music. Music provides direct links to important concepts including period, frequency, amplitude, ... sinusoidal signals.....”[2]

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This paper describes a project at Tri-cities High School in Atlanta, GA [3], conducted as part of Georgia Tech's Student and Teacher Enhancement Partnership (STEP) GK-12 program, to inspire and motivate students, and help them relate in-class mathematical concepts to real-world events. The primary strategy was to develop a series of computer laboratories using Mathwork's Matlab<sup>®</sup> software to help students relate to trigonometry and pre-calculus concepts through a "hands-on" multimedia approach. Specific topics to be covered were drawn from the trigonometry textbook [4] and supported the course curriculum. The labs ranged from computer adaptations of textbook exercises, such as using matrix rotations to rotate a star, to adaptations of college sophomore-level Digital Signal Processing laboratories. Two of these latter labs, entitled "Sine waves, Harmonics, and Music", inspired by [5], and "Communication Systems", inspired by [6], are included in the second section of this paper, *Exploring Sinusoids through Sound and Telecommunications*. The first section describes a lab that illustrates piecewise linear functions through a loudspeaker example, and the third section presents a lab illustrating statistical operators on images. All labs from the first half of the semester can be seen in [7].

### **STUDENT AND TEACHER ENHANCEMENT PARTNERSHIP (STEP) PROGRAM**

In 1999, the National Science Foundation initiated a new type of graduate student support through the NSF Graduate Teaching Fellows in K-12 Education (GK-12) program. Students receiving GK-12 fellowships are required to interact directly with K-12 teachers in an attempt to improve both K-12 education and the pedagogical and communication skills of the Fellows. In return, graduate Fellows receive an annual stipend and a tuition waiver. In the spring of 2001, Georgia Tech received a GK-12 grant to support its Student and Teacher Enhancement Partnership (STEP) program [8] and to place twelve graduate students per year in Atlanta area high schools. In 2004 the program received renewed funding for an additional five years.

The broad goals of the GK-12 initiative and of the STEP program are:

- (1) To broaden the education of science, technology, engineering, and mathematics (STEM) graduate students to include intensive experiences in educational pedagogy and process;
- (2) To encourage the participation of STEM faculty and students in the difficult issues facing K-12 educators through the nurturing of university-school partnerships;
- (3) To assist K-12 teachers in their endeavor to improve classroom instruction and improve K-12 student achievement in STEM.

To address these goals, STEP forms partnership teams at each of six metro-Atlanta high schools that consist of two Georgia Tech graduate STEP Fellows, a teacher STEP coordinator, and additional teachers and administrators from the school. Each STEP team then designs an action plan for the year based on the needs of the school, and the talents and interests of the particular STEP Fellows. During the 2003-2004 school year one STEP Fellow, Gail Rosen, directly assisted two Tri-Cities High School mathematics teachers in developing and implementing computer-based approaches for mathematics education by using her knowledge of electrical and computer engineering to demonstrate specific principles in trigonometry and pre-calculus. The laboratory exercises that follow are the result of this collaboration between an engineering graduate student and high school mathematics teachers.

Tri-Cities High School is a primarily minority, fine and performing arts magnet school (85% African American, 8% Hispanic) in south Fulton County with a 43% free/reduced lunch rate and 60% graduation rate. In 2003 64% of the students failed the science portion of the Georgia High School Graduation Test, and 25% failed the math portion. The many challenges faced during the implementation of each lab will be addressed in the paper discussion.

## TRIGONOMETRY AND PRE-CALCULUS COMPUTER LABORATORIES

### Lab Exercise #1--Demonstrating Piecewise Functions with a Loudspeaker Example

One topic covered early in the Trigonometry and Pre-Calculus classes was **piecewise linear functions**. To whet the students' appetites and give them a real-world example, the first session of the semester was an audio demonstration of the "clipping" phenomenon. The materials needed for this exercise were a computer with Matlab®, computer speakers, and a sound-clip of music.



Fig. 1: Loudspeaker

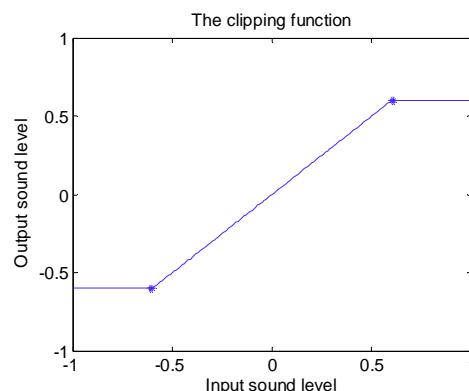


Fig. 2: Hypothetical piecewise clipping function

A loudspeaker produces sound with vibrating cone(s), the circular portions of Fig. 1. One example of clipping occurs when the cone of a speaker cannot extend out to the extent required by the high signal amplitudes (due to the old-age/rigidity, poor design of the cone, etc.). Therefore, loud sounds produced by the loudspeaker become distorted. Other factors in an audio system can contribute to this effect. For example, if a microphone has a certain range and a person gets too close or speaks too loudly for that range, clipping will occur. Whether it be in the school auditorium with an ill-configured PA system, getting too close to a microphone, or blasting music from a car stereo system, most young people have heard and been disturbed by the clipping phenomenon.

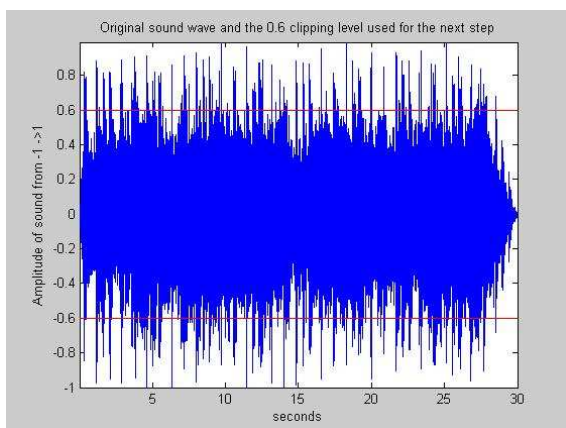


Fig. 3a: 30 second sound-bite.

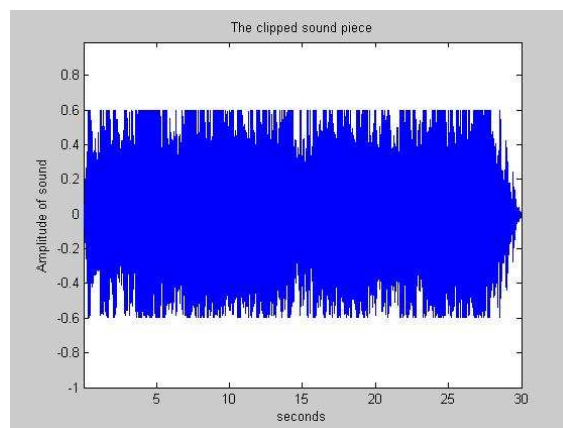


Fig. 3b: 30 second sound-bite after clipping.

Since this is an example that will hit home with many students, we developed a demonstration of the effect and tied it to the piecewise linear function concept. We can show the effects that the clipping function in Fig. 2 has on a 30 second sound-clip of music shown in Figure 3a; the result is seen in Figure 3b.

**Concepts Learned**

After the demonstration, the students were asked to write the piecewise function in mathematical form. The desired function was,  $y =$

$$\begin{array}{ll}
 -0.6 & -1 \leq x \leq -0.6 \\
 x & -0.6 < x < 0.6 \\
 0.6 & x \geq 0.6
 \end{array}$$

This demonstration took them back to the curriculum and reinforced a required mathematical concept, but students also gained a new appreciation for the applications of the mathematics, connecting the math back to reality and to audible and visual effects.

**Lab Exercise #2: Exploring Sinusoids through Sound and Telecommunications**

The following is an excerpt from the written communications lab activity:

*“Whether you use your cellphone or internet, you are sending information to a distant place. The information can be your voice when using the phone or letters when using the internet.*

*Say you want to send a Yes or No to another location. How many symbols would you need for such a signal? In computers, this is usually represented as a 1 or 0 (True/False). How would you assign numbers to Yes/No/Maybe?*

**How many symbols are present in the English language?**

*Imagine, I am now communicating information to you with combinations of this many symbols!*

*The cellphone and internet are examples of communication systems. Engineers had to come up with ways to encode information such as letters using even simpler basic blocks. (You can think of letters as the basic blocks of our written language).”*

The statements in **bold** are graded, but the rest of the introduction is used to inspire inquiry into telecommunications and how information is represented.

In the curriculum, students learn that the equation of a sine wave is  $y(t) = A \sin(2\pi f_c t)$  where  $A$  is the amplitude,  $f_c$  is the frequency, and  $t$  is the independent time variable. The second exercise in the lab is to explore how these sine waves can add to approximate a square wave, as illustrated in Figure 4. The materials needed for this exercise were a computer with Matlab®, Matlab® Simulink, computer speakers, and computer speakers.

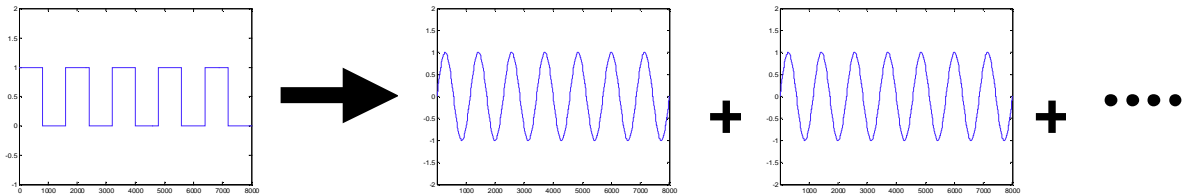


Fig. 4: A square wave is an approximation of infinite sine waves.

Fig. 5a shows a simulink model where four sine waves can be added to approximate a square wave. The following equation, a truncated periodic square wave Fourier series, was used:

$$y(t) = 1 * \sin(2\pi f_c t) + \frac{1}{3} \sin(2\pi * 3 * f_c t) + \frac{1}{5} \sin(2\pi * 5 * f_c t) + \frac{1}{7} \sin(2\pi * 7 * f_c t) .$$

The students observed that as they added the sine wave components, the wave approached a square wave. They then listened to the sine wave and noted that it sounded harsher than the sine timbre; timbre is the quality of the sound or tone color. Although, they may not understand why the waveform changes as they add sines, it introduces them to the Fourier series concept and sparks curiosity about what other waveforms they can make by adding them together.

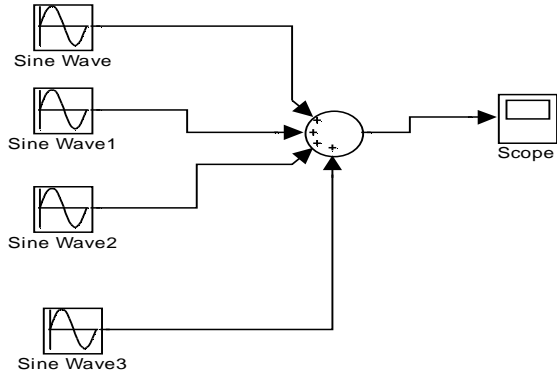


Fig. 5a: Simulink model of a 4 sine wave adder

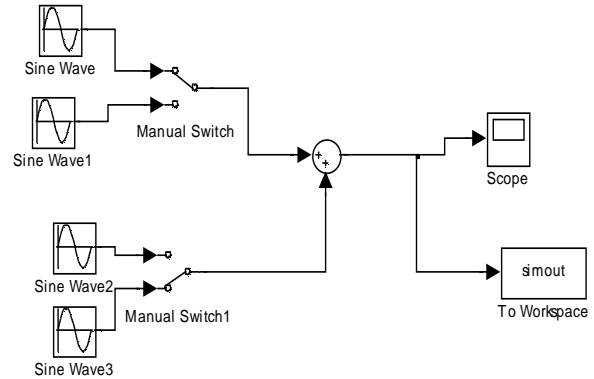


Fig. 5b: Simulink model of a DTMF dialer

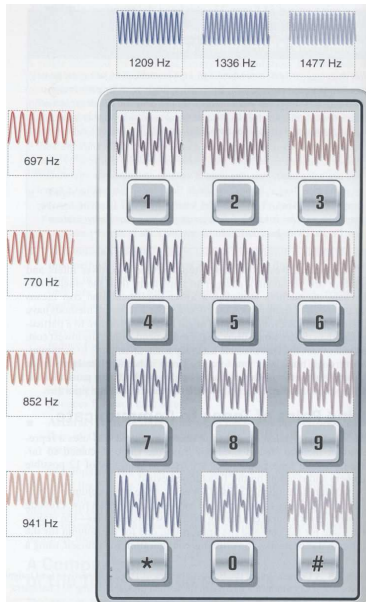


Fig. 6: DTMF telephone keypad [9]

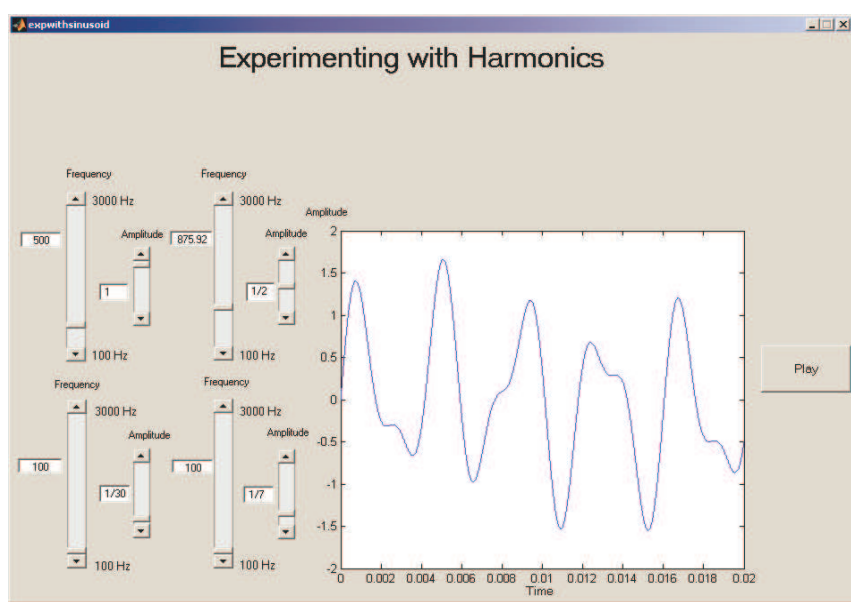


Fig. 7: Graphical user interface for students to adjust amplitude and frequency parameters of added sine waves.

In the final section of the lab, the concepts of adding sines together was connected with the concept of transmitting information. We introduced DTMF (Dual-Tone Multi-Frequency) dialing, or what is more commonly known as touch-tones. The students made the dialer, shown in Fig. 5b, in Simulink. They then could see the results in the scope as well as play the simout sounds. By switching different sine waves through and combining two of them, they could make any tone. The students then counted the different tones and for extra credit, calculated how many combinations there would be for three sine waves; this reinforced the concept that these combinations can represent different “symbols”, illustrated on the DTMF telephone keypad (Fig. 6).

An extension of this laboratory was used in a subsequent laboratory where a user interface, in Fig. 7, was designed so that the students could adjust the amplitude and frequency of each sine wave quickly and play the result. They

were encouraged to write down the equation they were playing, what it sounded like, and note any observations in trends.

**Concepts Learned**

The students learned how communication systems use math to represent symbols, and specifically how telephone touch-tones are created. Also they learned how addition of sinusoids can make different waveforms and sound. Those students with a musical background connected better to the exercise because they already understood the concepts of harmonics and how they affect timbre, so they seemed to catch on how to alter the equation mathematically to make it sound like pleasing instruments.

**Lab Exercise #3: Image Noise Removal With Statistical Operations**



Fig. 8a: Original gray-scale image      Fig. 8b: Gaussian noise added      Fig 8c: Salt-and-pepper noise added

The students were given images Figures 8b and 8c. They were then given a Matlab<sup>®</sup> program that took a user-defined sized block that went pixel-by-pixel over the image and performed a **median** or **mean** operation on the blocked portion. Actually, the algorithm was designed slightly off – for each pixel, it looked “ahead” by a determined number of pixels. This produced a substandard result; for extra credit, students were encouraged to offer solutions on how to improve it. All the students were familiar with the fact that the mean is defined by

$$\frac{1}{N} \sum_{i=0}^N x_i$$

where  $N$  is the amount of numbers and  $x_i$  is each number in the sequence, and that the median is

approximated as the  $x_{\text{round}(N/2)}$  of a sequence. They obtained the following results for the image with added Gaussian noise (Fig 9a-d), and the image with added salt and pepper noise (i.e. snow) (Fig 10a-d) :

**Processing on the noisy image**

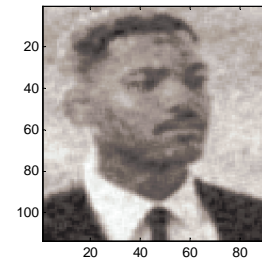
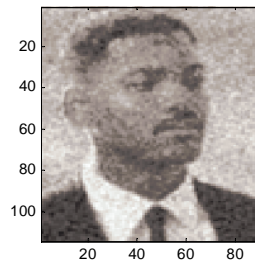
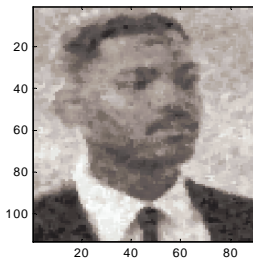
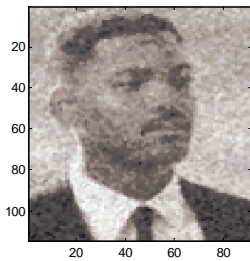


Fig. 9a: 2x2-block median      Fig. 9b: 4x4-block median      Fig. 9c: 2x2-block mean      Fig. 9d: 4x4-block mean

**Processing on the snowy image**

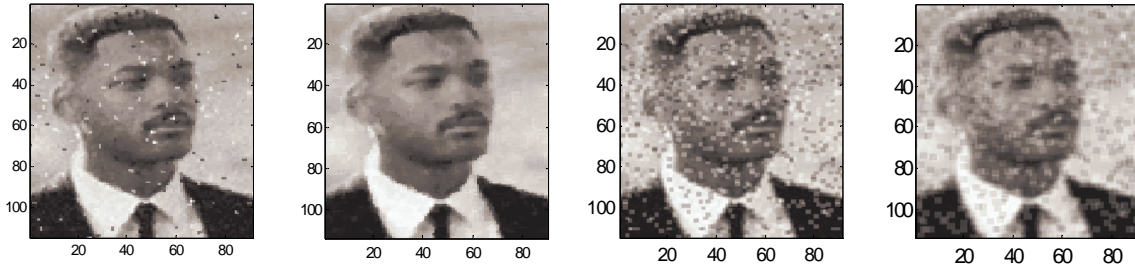
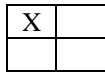


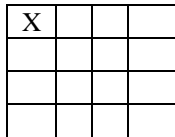
Fig. 10a: 2x2-block median Fig. 10b: 4x4-block median Fig. 10c: 2x2-block mean Fig. 10d: 4x4-block mean

**Concepts Learned**

The students learned two major concepts about how these functions “cleaned” the image. One was that the larger the block size, the more the noise was smoothed, but the image was also. This resulted in the major concept of noise removal vs. blurring trade-off. The second concept was that the mean worked better for the *noisy* (Gaussian) image while the median worked better for the *snowy* (salt-and-pepper noise) image. Even though the snowy image looked the worst, a 4x4 pixel block was able to remove all the noise and with minimal blurring. For extra credit, the students explored how they were doing the image cleaning:

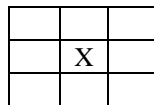


2x2 block



4x4block

(For each block, the X indicates which pixel the mean/median modified on each iteration.) Students were then asked how they thought they should modify the pixels to get better results. The answer was:



Odd order block with the median/mean updating the middle pixel value.

And if they had time, students implemented this change, though it was beyond the scope of the lab.

This was an intensive lab because although the students were not required to write any Matlab<sup>®</sup> code, they needed to figure out how to alter the parameters so they could change the block size, and they also had to evaluate the results for their conclusions. It is difficult sometimes for students to comprehend that a method doesn't work 100%. The students needed to explain the trade-off that the *snowy* image was cleaned of the noise yet became blurred, a task that required analysis skills.

One student who hadn't really excelled in any of the other labs, flew through this one and really was fascinated at the possibility that he could use Matlab<sup>®</sup> to alter images or perhaps even remove scratches in a picture he had. Even though we had focused on sound and sine waves earlier in the semester because it more closely related to the curriculum, it is worthwhile to attempt image analysis/synthesis labs to reach out to the more visually-inclined learner.

## CHALLENGES IN IMPLEMENTING COMPUTER LABS

The biggest challenge in implementing computer labs at a low income high school was the state of the computer lab, which was used intensively (though not necessarily productively) by all the math classes at the school. The copies of Matlab<sup>®</sup>, generously donated by Mathworks, were the student version, which meant they each needed the documentation CD to operate. We also required working computers, with working accessories such as the mouse and monitor, as well as functioning speakers (for the audio lab) and CD-ROM drives. The computer speakers added the excitement to the audio laboratory but were not absolutely essential. As the year progressed, parts began to break on the computers – monitors would stop working, mice would disappear, and CD-ROMs would especially go bad since students excessively used them to play audio CDs. The lack of computer speakers in the lab caused students to “borrow” them from other computers – this resulted in the speakers playing “musical computers”. You never knew where they would be hooked up in the lab or if they were hooked up properly. This was a constant source of frustration for all involved, because when a computer didn’t work, it delayed lab time or students would need to put into groups of 3 instead of 2, which would ultimately decrease learning and productivity. A server version of Matlab<sup>®</sup>, as well as sufficient computer supplies and backups, and proper maintenance of the labs, would have made the implementation much easier.

The major challenge with the auditory labs was that when the computers, CD-ROMs, and speakers were functioning properly, students were too easily tempted to play audio CDs and mp3’s or to play internet games instead of completing the lab. Each teacher placed a certain grade value on the computer labs, and the less each was worth, the more likely the students were to be tempted to diverge when completing the auditory labs. From the outset, the instructor should place restrictions on the computer speaker use.

Another lesson learned was to not make the labs fully inquiry-based. The lab instructions always contained an explanation of the lab protocol and theory, but most students would not read this and would then complain later that they didn’t understand what the lab was about. Theoretically, students could learn the concepts through action and experimentation, but this generally did not occur. Being only task-driven, the students completed the steps but took no time to understand the concepts. A 15-minute lecture at the beginning explaining the concept and the connection to the classroom curriculum of the lab would have improved student learning.

## STUDENT FEEDBACK

The following are quotes from student surveys.

*“I really enjoyed doing the labs and working with the computers. We got a chance to work with sine waves and learn how they are used in our everyday lives. Today we worked with Matlab and learned how dial tones on our phones are made. Using Matlab and the computers opened my eyes to how many aspects of my everyday life are centered around technology and computers.”*

*“I thought Ms. Rosen’s labs were very interesting. The activities that she had us doing helped me as a musician to understand the science of music. My only suggestions is that these labs should be focused on more students who are interested in the content discussed.”*

*“Overall all of the labs we’ve done in matlab has some what beneficial to it. It has helped me relate calculus and other math subjects to real life. I really didn’t know this much math was in our everyday lives. Since I may major in a science field when I go to college some of this experience may help me in the future. Thanks Ms. Rosen!!!”*

*“I think her labs were great! They gave me a lot of insight on sound and how they form waves! I could honestly say my favorite lab was the sound lab when we tested for all types of*



*sound waves! I really didn't have any labs I didn't like! I think she can add more image and sounds labs, though!"*

*"Truthfully, I believe that these labs has only good for those who sought purpose. They were all good learning experiences and it was good to know. Sometimes it got confusing and frustrating and that was the difficult part about it. Thank you for your time."*

## SUMMARY

In conclusion, three pre-calculus and trigonometry labs were introduced that highlight the relation of math with sound and images. The students learned how piecewise linear functions and sinusoids relate to music and sounds in their everyday life. Also, they learned how an image could be "cleaned" from noise and artifacts. There are also many real-world lessons for the instructor in implementing computer labs, such as minimizing the equipment needed, making sure there is little time for distraction in auditory-based labs, and that lecture introductions are needed in addition to inquiry-based labs. Also, end-of-the-semester evaluations validated effectiveness of the computer laboratories to motivate students.

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**Gail Rosen**

Gail Rosen received both a B.S. (highest honors) and M.S. in electrical engineering from the Georgia Institute of Technology, in 1999 and 2002, respectively. Since 2002, she continues at the Georgia Tech's Center for Signal and Imaging Processing (CSIP) to pursue a Ph.D. in electrical engineering. She is a recipient of numerous awards including an NSF Graduate fellowship (2000), an NSF STEP fellowship (2003), an AT&T Research Laboratories grant (2000), and a Georgia Tech ECE outstanding teaching award (2003). Her main research interests are reverse-engineering biological systems and analyzing DNA structure, mutations, and repair.

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