Controllability of Magnetic Manipulation of a Few Microparticles in Fluids

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Control theory is employed to formulate the problem of controllability of two microparticles in fluids via magnetic field. We demonstrate that a uniform external magnetic field of varying direction and magnitude provides complete local state controllability of two particles over a magnetized substrate. We propose that such an approach may improve manipulation and assembly of microparticles in fluids.

***Index Terms*—Controllability, magnetic manipulation, tracking.**

I. INTRODUCTION

MASSIVELY parallel assembly and manipulation of colloidal objects in fluids is a promising methodology for fabrication of heterogeneous microsystems, for sorting and analysis of biological objects and for many other applications. Some aspects of assembly and manipulation of magnetic and nonmagnetic microparticles have been recently demonstrated experimentally [1], [4]. However, many important questions regarding this methodology remain open. One important issue, which seems to have been largely ignored, is the controllability of the parallel magnetic assembly and manipulation. In such applications, one is most interested in controlling a number of colloidal magnetic objects using the smallest possible number of external parameters. For example, it would be convenient to control positions and movements of several particles using only a uniform external magnetic field.

In this paper, we consider the simplest system of interest containing two spherical magnetic microparticles in fluid and a permanently magnetized micromagnet, as illustrated in Fig. 1. The field produced by the micromagnet is approximated as the field due to a single isolated magnetic pole. The spherical microparticles are treated as linearly magnetizable dipoles. The evolution of their positions representing the internal state of the system is described by a set of coupled first-order differential equations. The external uniform magnetic field is viewed as the system's input. Since the relation between the state and the input in this system is nonlinear and its controllability is difficult to analyze, we focus on a simpler problem of local controllability as a starting point. Local controllability can be used to predict the possibility of achieving any desired small deviation from particle trajectories obtained for fixed external fields. Such a property can be particularly interesting near positions of unstable equilibria as it would permit us to steer particle assembly into desired configurations. Local linearization is employed to describe the evolution of particle's state around any given initial state in order to investigate the possibility of local controllability. The controllability matrix for this locally linearized system is derived analytically and evaluated at various initial states.

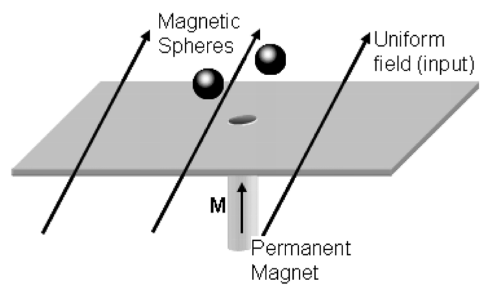

Fig. 1. Magnetic assembly and manipulation system illustration.

Using this formalism, we observe a somewhat counterintuitive result that manipulation of two particles can be controllable in the vicinity of the micromagnet with only a uniform external magnetic field of fixed direction. This means that 6 degrees of freedom representing particle positions can be controlled by an input with only 1 degree of freedom.

II. MODEL DESCRIPTION

A. Coupled Equations for Evolution of Particle's Positions.

The model described in this paper is intended to provide simplified guidelines for controlling the assembly of interacting (100 nm–100 μm) superparamagnetic particles onto surfaces using the minimum number of external inputs. In the model, particles are treated as point dipoles whose magnetic moments $\vec{m}$ are proportional to the fields at their centers according to

$$\vec{m}_1 = \frac{3\chi V}{3+\chi}\vec{H}_1 = s\vec{H}_1, \quad \vec{m}_2 = \frac{3\chi V}{3+\chi}\vec{H}_2 = s\vec{H}_2 \qquad (1)$$

where χ is the particle's susceptibility, V is their volume, and $\vec{H}$ is the total magnetic field which is the sum of the uniform magnetic field $\vec{U}$, the particle interaction field $\vec{H}_{12}$, and the micromagnet's field $\vec{h}$. Here, we ignore the effect of size and domain pattern of the micromagnet and approximate its field as the field due to an isolated point magnetic pole located at the origin

$$\vec{h}(\vec{R}) = \frac{q}{4\pi}\frac{\vec{R}}{R^3} \qquad (2)$$

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where q is the pole strength and $\vec{R}$ is the position vector of a point where the field is evaluated. The particle's magnetic interaction fields are given by

$$\vec{H}_{kn} = \frac{1}{4\pi}\left(\frac{3(\vec{m}_k \cdot \vec{R}_{kn})\vec{R}_{kn}}{R_{kn}^5} - \frac{\vec{m}_k}{R_{kn}^3}\right) \tag{3}$$

where indexes k and n represent particles 1 and 2 interchangeably and the first index indicates the particle responsible for the field, while $\vec{R}_{kn} = \vec{R}_n - \vec{R}_k$ is the vector from the kth particle to the nth.

The magnetic force on each particle is now derived by the following differentiation, where magnetic moments are treated as constants:

$$\begin{aligned}\vec{F}_n &= \mu_0(\vec{m}_n \cdot \nabla_n)\vec{H}_n \\ &= \frac{\mu_0}{4\pi}(\vec{m}_n \cdot \nabla_n)\left(\frac{3(\vec{m}_k \cdot \vec{R}_{kn})\vec{R}_{kn}}{R_{kn}^5} - \frac{\vec{m}_k}{R_{kn}^3} + \frac{q\vec{R}_n}{R_n^3}\right)\end{aligned} \tag{4}$$

where ∇_n denotes the gradient operation with respect to the position of the nth particle and μ_0 is the permeability of free space. The particle's magnetic moments are obtained self-consistently given the external uniform field and the micromagnet's field from the following two coupled linear equations derived from (1)–(3):

$$\vec{m}_n = s\left[\frac{1}{4\pi}\left(\frac{3(\vec{m}_k \cdot \vec{R}_{kn})\vec{R}_{kn}}{R_{kn}^5} - \frac{\vec{m}_k}{R_{kn}^3}\right) + \frac{q}{4\pi}\frac{\vec{R}_n}{R_n^3} + \vec{U}\right]. \tag{5}$$

The equation of motion for each particle is obtained by neglecting the particle's acceleration and equating the magnetic force on it to the fluid drag force

$$F_{\text{drag}}(\vec{R}_n) = 6\pi\eta\, a\frac{\partial \vec{R}_n}{\partial t} = \vec{F}_n \tag{6}$$

where a is the particle's radius, η is the dynamic viscosity of the fluid, and Stoke's approximation for the drag force on the particle's is employed.

Equation (6) represents two coupled first-order differential equations ($\mathrm{n} = 1, 2$) which describe the evolution of the particle's positions. The coupling of these equations is due to magnetic interactions described by (4) and (5). Although hydrodynamic interactions with walls and between particles can play a role in particle motion, they are ignored in this simplified model.

B. Local Linearization and Controllability

In general, controllability requires existence of input $\vec{U}$ as a function of time that can take the system from any initial state (position in this case) to any final state within finite time. Proving the existence of such an input for nonlinear systems, such as the one considered here, is challenging. In this paper, we first address a simpler question: Can small deviations from a given particle trajectory be controlled? The linearization of (6) is obtained to answer this question. The controllability of the resulting linear system is a much simpler question that can be addressed using the well-known formalism of linear control theory.

Denoting small deviations in the particle's position by $\vec{r}_n$ and small changes in the input by $\vec{u}$, the following coupled differential equations can be obtained from (6) by neglecting terms of order higher than $\vec{r}_n$ and $\vec{u}$:

$$\frac{d\vec{r}_n}{dt} = \left[\frac{\partial \vec{F}_n}{\partial \vec{R}_n}\right]\vec{r}_n + \left[\frac{\partial \vec{F}_n}{\partial \vec{R}_k}\right]\vec{r}_k + \left[\frac{\partial \vec{F}_n}{\partial \vec{U}}\right]\vec{u} \tag{7}$$

where $[\partial \vec{F}_n/\partial \vec{R}_n], [\partial \vec{F}_n/\partial \vec{R}_k]$, and $[\partial \vec{F}_n/\partial \vec{U}]$ can be viewed as matrices whose entries are derivatives of the Cartesian force components with respect to various Cartesian components of vectors $\vec{R}_n, \vec{R}_k$, and $\vec{U}$, respectively. The above equation can be written in a matrix form as follows:

$$\frac{d\vec{r}}{dt} = \hat{A}\vec{r} + \hat{B}\vec{u} \tag{8}$$

where $\vec{r} = [\vec{r}_1, \vec{r}_2]^T$ is a column vector of all Cartesian components of the particle's positions and

$$\hat{A} = \begin{bmatrix} \left[\frac{\partial \vec{F}_1}{\partial \vec{R}_1}\right] & \left[\frac{\partial \vec{F}_1}{\partial \vec{R}_2}\right] \\ \left[\frac{\partial \vec{F}_2}{\partial \vec{R}_1}\right] & \left[\frac{\partial \vec{F}_2}{\partial \vec{R}_2}\right] \end{bmatrix} \quad \text{and} \quad \hat{B} = \begin{bmatrix} \left[\frac{\partial \vec{F}_1}{\partial \vec{U}}\right] \\ \left[\frac{\partial \vec{F}_2}{\partial \vec{U}}\right] \end{bmatrix}. \tag{9}$$

It is well known that system (8) is controllable if and only if the following controllability matrix is full rank [2]:

$$\hat{C} = [\hat{B}\ \hat{A}\hat{B}\ \hat{A}^2\hat{B}\ \cdots\ \hat{A}^{N-1}\hat{B}] \tag{10}$$

where N is the number of independent degrees of freedom the state has. In our case, $N = 6$. Since the controllability matrix is evaluated numerically, we use its condition number instead of its rank to determine controllability. The condition number p is defined as the ratio of the largest $\omega_{\max}$ to the smallest $\omega_{\min}$ singular values of the matrix, i.e.,

$$p = \frac{\omega_{\max}}{\omega_{\min}}, \quad \omega_{\max} = \max\sqrt{\text{eigenvalue}\,(\hat{C}\hat{C}^T)},$$

and

$$\omega_{\min} = \min\sqrt{\text{eigenvalue}\,(\hat{C}\hat{C}^T)}. \tag{11}$$

When the condition number is substantially larger than ten, the system is difficult to control.

III. Results and Conclusion

A. Controllability Measure

In simulations, the magnetic particles were assumed to have magnetic susceptibility of 2.5, a typical value for commercially available magnetic particles [3] and the diameter of 100 μm. In calculating the magnetic pole strength $q = \Omega^{D_r}/\mu_0$, the permanent micromagnet was assumed to have a cross-section area Ω of 100 by 100 μm and a remnant magnetic flux density D_r of 1.0 Tesla.

The controllability matrix $\hat{C}$ in (10) was evaluated numerically using analytical expressions for matrices $\hat{A}$ and $\hat{B}$ obtained

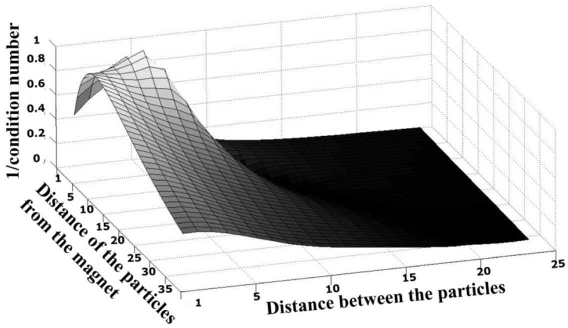

Fig. 2. Controllability map of two-particle system when particles remain above permanent magnet at height of one radius. External field is applied in y direction. Distances are measured in 0.2 particle radii.

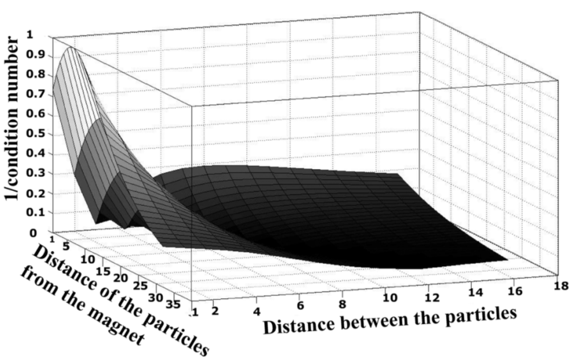

Fig. 3. Controllability map of two-particle system when particles remain above permanent magnet at height of 1 radius. External field is applied in x direction. Distances are measured in 0.2 particle radii.

using (4) and (5). In doing so, (5) is solved and used for evaluation of force in (4). The resulting analytical expressions are far too lengthy to be explicitly included in this short publication. However, they were confirmed using symbolic algebra in Maple software and by numerical evaluations of the derivatives.

The controllability matrix was evaluated this way over a mesh of different particle positions and for different directions and values of the uniform magnetic field. Examples of the maps of the inverse of the condition number for the controllability matrix are shown in Figs. 2–4. It was observed that the controllability matrix was full rank and the condition number was below 20 when both particles were within about 3–4 diameters away from the magnet. This indicates that controllability is achieved due to the interaction between the particles. Such a conclusion agrees with the intuition that two independent particles cannot be controlled using the same external uniform field. At the same time, it is remarkable and not at all obvious *a priori* that controllability can be achieved through particle interaction.

Interestingly, as illustrated in Fig. 2, the condition number behaves in a nonmonotone fashion as a function of distance between the particles. If we accept that particle interaction is needed for controllability, this nonmonotone behavior also agrees with intuition. Indeed, when particles are far away from each other, controllability should be poor. On the other hand,

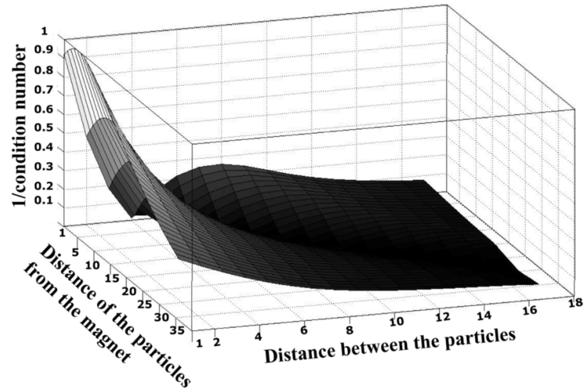

Fig. 4. Controllability map of two-particle system when particles remain above permanent magnet at height of four radii. External field is applied in y direction. Distances are measured in 0.2 particle radii.

when the distance between the particles is much smaller than their average distance to the micromagnet, both particles are subject to nearly identical total fields and behave as one. For this reason, controllability over 6 degrees of freedom is lost again. Another interesting point revealed by the simulations is that the magnitude of the uniform field should be on the order of the field experienced by the particles due to the micromagnet. This is natural because, when the uniform field is much larger than the micromagnet field, the particle's magnetic moments align mostly along the uniform field and cannot be tuned separately.

In conclusion, we demonstrate that the movement of two particles in fluids in the presence of a micromagnet can be locally controlled via a uniform external magnetic field. Such local controllability can be used to correct particle movement and may be particularly important around positions of zero magnetostatic force. It is not yet clear whether this controllability theory can be extended to more than two particles. However, since the source of the enhanced local controllability has been demonstrated to be the magnetic interactions amongst the particles, we expect our results to generalize for a large number of particles. Future research is being undertaken to answer this question.

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