The simplest index coding problem requiring a non-Shannon inequality

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Outline

- Non-Shannon inequality review
- Index coding review
- Index coding in interference alignment perspective
- Simplest multiple unicast index coding problem requiring non-Shannon
- Extensions and discussions

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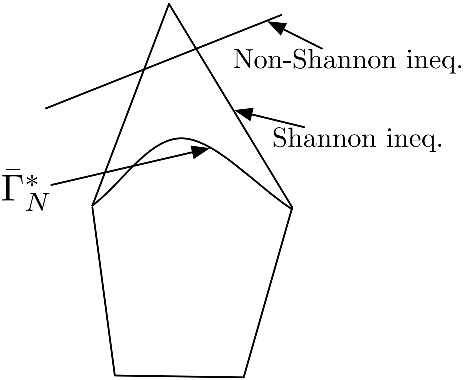
Non-Shannon inequality

- Region of entropic vectors $\bar{\Gamma}^*_N$
- Natural outer bound: Γ_N formed by Shannon-type inequalities, $I(i,j|K) \geq 0$ where i, j are random variables and K is a collection of random variables. $i = j$ and $K = \emptyset$ are allowed.
- Zhang-Yeung non-Shannon type ineq. found in 1998 [3]
- Non-Shannon ineq. is derivable from inequalities define Γ_N (but is derivable from ineq. for $\Gamma_{N'}, N' > N$)



Zhang-Yeung non-Shannon ineq.

- Shannon type ineq. obey non-negativity of conditional mutural information and polymatroid axioms: monotonicity and submodularity
- First non-Shannon type ineq. found [3]
- For 4 variables $\{A, B, C, D\}$, we have $I(C;D) - I(C;D|A) - I(C;D|B) \leq \frac{1}{2}I(A;B) + \frac{1}{4}[I(A;CD) + I(B;CD)]$
- Which is equivalent to $3H(A,C) + 3H(A,D) + 3H(C,D) + H(B,C) + H(B,D) \geq 2H(C) + 2H(D) + H(A,B) + H(A) + H(B,C,D) + 4H(A,C,D)$

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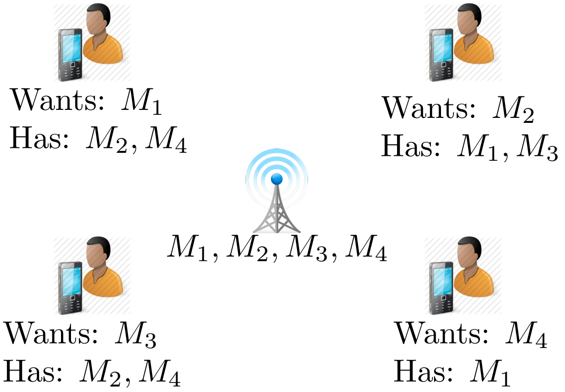
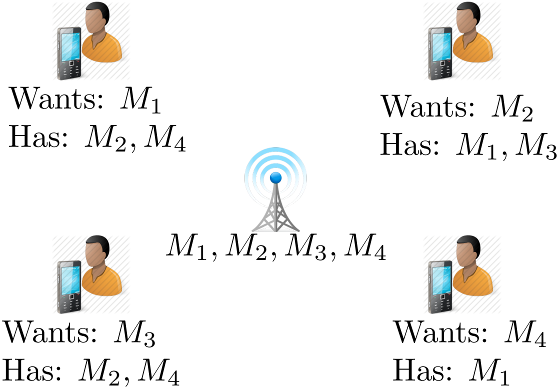
Index coding (IC) motivation

- A set of packets need to be delivered to a set of users
- Users may have some information already (side information)
- Broadcast channel is used by the server
- Goal: minimum transmission times



Index coding motivation

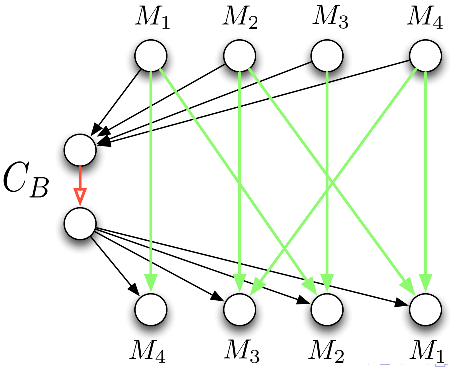
- Traditional way: send M_1, M_2, M_3, M_4 respectively
- Coding: send $M_2 + M_3, M_1 + M_4$ respectively
- Side information is utilized
- Transmission times are reduced



Index coding model

The above example can be represented as a typical index coding model:

- Sources, sinks with requirements and side information, broadcasting node with coding
- Broadcast link has capacity C_B, others unlimited
- Symmetrical capacity R at source side can be normalized by C_B



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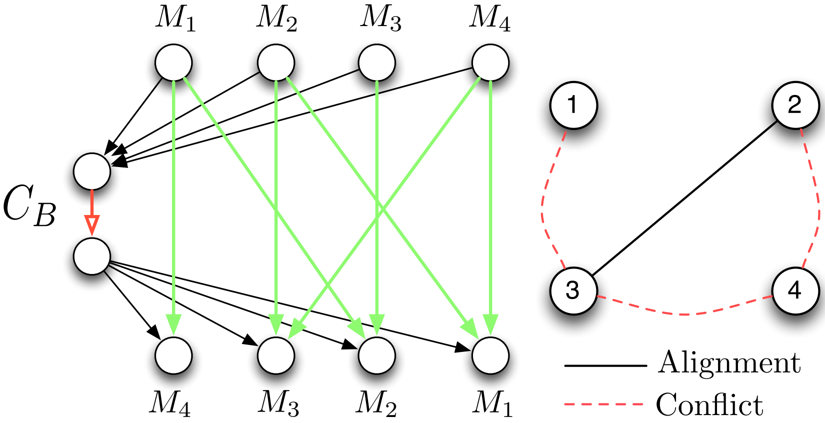
Interference alignment

- Interference: received messages that are not demanded by the receiver
- Alignment: all interferences should occupy as small a signal space as possible so that we have enough space to decode the desired messages
- Alignment graph: every message is represented as a node, connect two nodes if these two are interference for one receiver
- Conflict graph: connect a desired message and its interferences since they are conflict

Alignment and conflict graph for index coding

- Alignment: (2,3)
- Conflict: (1,3), (3,4), (2,4)



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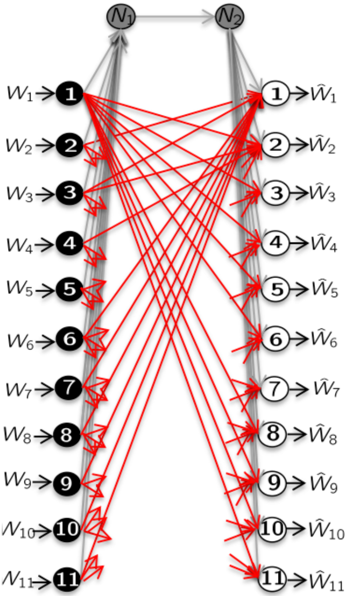
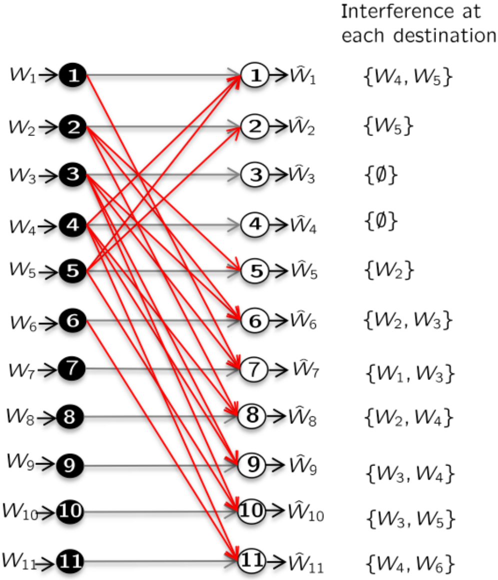
Simplest multiple unicast requires non-Shannon

- Construction [1]: 11 sources, and 11 unicast pairs, each receiver has no more than 2 interference
- Red arrows show side information (antodotes)



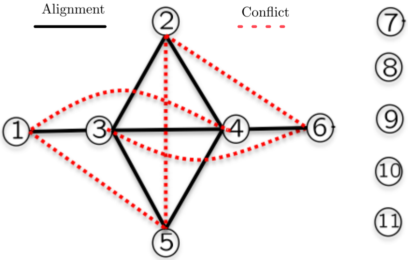
Simplest multiple unicast requires non-Shannon

- Red arrows show interference links



Simplest multiple unicast requires non-Shannon

- Alignment and conflict graph
- Minimum conflict distance Δ: minimum number of edges in the alighment graph that conflict edges cover
- For this example: $\Delta = 2$



Simplest multiple unicast requires non-Shannon

- Since $\Delta = 2$, according to [2], the upper bound on the (normalized) capacity $R = \frac{\Delta}{2\Delta+1} = \frac{2}{5}$
- If Zhang-Yeung non-Shannon ineq. is used, this upper bound can be tighter to $R = \frac{11}{28}$
- Sketch of proof:
  - Give explicit assignments for the variables with $R = \frac{2}{5}$ and show that the assignments obey polymatroidal axioms (the assignments can be imagined as vector (entropy) space assignments)
  - Use Shannon ineq. to get lower bound on dimensions occupied by the entropy space of the triangle (2,3,4) and (3,4,5) in the alignment graph (Fano's ineq. is used here)
  - Then consider the diamond (2,3,4,5) in the alignment graph, apply Zhang-Yeung ineq. to obtain an upper bound for (2,3,4) and (3,4,5)
  - Combine these two pieces, get an ineq. on R

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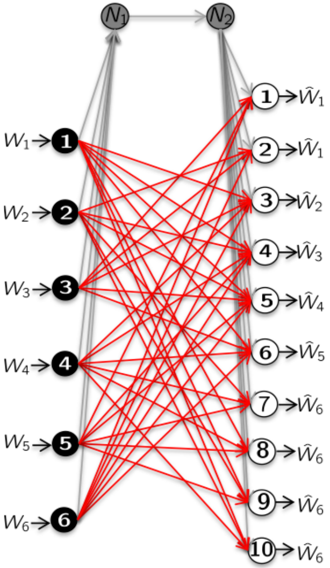
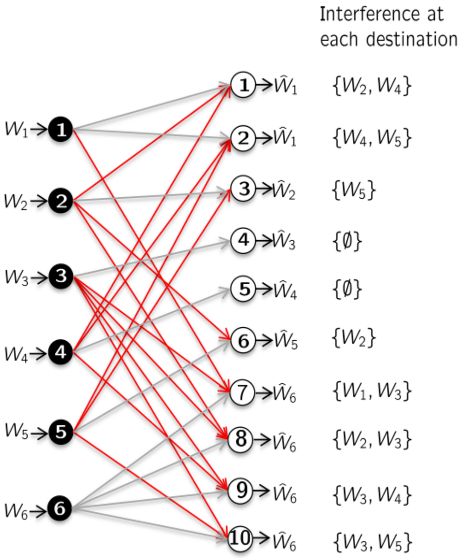
Simplest multiple groupcast requires non-Shannon

- Construction [1]: 6 sources, and 10 receivers, each receiver has no more than 2 interference
- Red arrows show side information (antodotes)



Simplest multiple groupcast requires non-Shannon

- Red arrows show interference links



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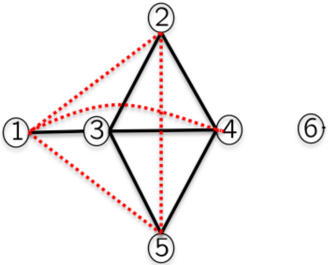
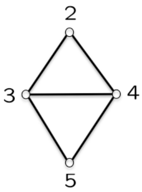
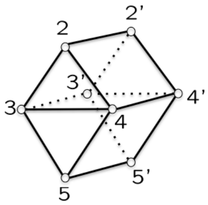
- For this example: $\Delta = 2$



Compare with V́amos matroid

- Geometry representation of V́amos matroid:
  - rank is 4
  - any 4-element subsets are independent except $\{2, 2', 3, 3'\}$,$\{2, 2', 4, 4'\}$,$\{3, 3', 4, 4'\}$,$\{3, 3', 5, 5'\}$,$\{4, 4', 5, 5'\}$
- If let 2=(2,2'); 3=(3,3'); 4=(4,4'); 5=(5,5'), V́amos matroid is converted to the essential part in the alignment graph for the problem we discussed





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2. S. A. Jafar, "Interference alignment: A new look at signal dimensions in a communication network", in *Foundations and Trends in Communication and Information Theory*, 2011, pp 1-136
3. Z. Zhang, R. W. Yeung, "On characterization of entropy function via information inequalities", *IEEE Trans. Info. Theory*, vol 44, no. 4, pp. 1440-1452, Jul 1998.