

ECES 521, Homework 4

October 19, 2013

Name:

Student ID:

1. Alice and Bob play a sudden-death chess match whereby the first player to win a game wins the match. Each game is won by Alice with probability p , by Bob with probability q , and is a draw with probability $1 - p - q$.
 - (a) What is the probability that Alice wins the match?
 - (b) What is the PMF, the mean, and the variance of the duration of the match?

2. The lifetime of a machine (in days) is a random variable T with probability mass function $p_T(t)$. Given that the machine is working after x days, what is the conditional PMF of T , and what is the mean subsequent (i.e. remaining) lifetime of the machine when:
 - (a) $p_T(t) = (N + 1)^{-1}$ for $t \in \{0, 1, \dots, N\}$,
 - (b) $p_T(t) = 2^{-t}$ for $t = 1, 2, \dots$

3. A certain office furniture company buys and rents new couches. It can buy a single couch for \$1000, or, if it buys two couches, it can obtain them each at a reduced price of \$750 per couch. After a couch is rented and returned by a customer, it is sold as a used couch for \$300. The company has two potential customers. One customer, an investment bank, is willing to pay \$5 per day for a couch, but every day risks being shut down by the government permanently with a probability 1% and will only pay for the days it is open before it is shut down. The second customer, a hotel chain, will rent a couch for a period of 365 days, but will only use it each day of these 365 days with a probability 75% (independently of the other days), and offers to pay \$ 3 on days it uses the couch and only \$ 2 on days that it doesn't use it. If the office furniture company decides to make its decisions by maximizing its expected profit, how many couches will it buy, and who will it agree to rent the couches to?

4. You must keep your house lit by buying lightbulbs. You have two options, one lightbulb costs \$ 25 and fails each day (independent of the previous days) will probability 2%. A second lightbulb costs \$30 and fails each day with probability 1%. Which of the two lightbulbs will minimize your long run average daily cost $\frac{c}{\mathbb{E}[L]}$ (where c is the cost of the lightbulb and L is the lifetime of the lightbulb)?

5. The company you work for presently stocks a large fleet of identical sales cars, each of which, on each day, independently of the previous days, fail permanently with a probability .1%. Each of the cars cost \$20,000 when it buys them new. However, the company could decide to rent them from another company at \$ 20.01/day, in which case the cars are guaranteed to never fail when in service on the fleet, as the (other) rental company must replace them if they are about to fail. The company hires two analysts to help it decide whether to continue buying its fleet of cars, or to rent them.
 - (a) One analyst makes a spreadsheet in which he calculates and enters the daily cost of each car $\frac{c}{L}$ (where c is its cost and L is its lifetime in days). He then averages all of these numbers in the spreadsheet. If the company had an infinitely large fleet, what number would this analyst obtain? If this analyst compares this number with the daily cost of renting a car, what would they recommend?
 - (b) A second analyst makes a spreadsheet in which he saves the lifetime of each car. He then averages these lifetimes, and divides the cost of a single car by the average lifetime he obtained. If the company had an infinitely large fleet, what number would this analyst obtain? If this analyst compares this number with the daily cost of renting a car, what would they recommend?
 - (c) **Bonus** Based on the process they utilized, which analyst should you listen to, and why? (Use a law of large numbers.)