ECES 632 Homework 1

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ECES 632, Winter Quarter, 2009

1. The CDF for a mixed random variable which takes only a finite number of values with strictly positive probability is always **corlol** (continuous on the right with limits on the left)... or if you are a Francophone **càdlàg** (continu à droite, limites à gauche). Considering the definition of the cumulative distribution function, and limiting your argument to mixed (continuous and discrete) random variables whose cumulative distribution functions have a finite number of discontinuities, explain why this is the case. What is the meaning of the difference between the left and right limits at a discontinuity of the CDF?

2. Suppose X is an exponential random variable with parameter λ, and Y distributed independently from X according to a Gaussian distribution with an m and variance σ^2. Let V be the random variable defined by
$$V := |X - b| + Y^2$$
   (a) Are V and X independent?
   (b) Write the probability density for V. (You may include non-closed-form-calculated integrals in your answer).

3. Suppose $\mathbf{X}$ is a jointly Gaussian random vector. What is the probability density for $\mathbf{Y} := \mathbf{A}\mathbf{X} + \mathbf{b}$ for some given deterministic $M \times N$ matrix $\mathbf{A}$ and length M column vector $\mathbf{b}$. (Hint: exploit a special property of the jointly Gaussian distribution to answer this question). Remember to consider carefully different situations regarding M, N and the rank of $\mathbf{A}$.

4. Please answer the following related questions:
   (a) Under what condition (on the mean and covariance functions) is a Gaussian random process a Markov random process?
   (b) Give examples of at least two random processes having this property, one that is wide sense stationary and another which is not. What are the names of your two examples?

5. Suppose a discrete time white noise process is passed through a stable linear filter with rational system function
$$H(z) = \frac{B(z)}{A(z)}$$
   (a) Would the output of the system be wide sense stationary? Prove your answer.
   (b) What would be the power spectral density of the output?
   (c) Suppose now that we only knew the power spectral density of the output and we didn't know $H(z)$. Could we uniquely determine the frequency response $H(e^{j\omega})$ of the filter? If so, how? If not, why not?
   (d) Suppose now that we only knew the z-transform of the auto-correlation function of the output of the filter, and again we didn't know $H(z)$. How could one use a spectral factorization in this instance to make a guess at $H(z)$? Under what additional conditions on $H(z)$ would this guess yield $H(z)$ exactly?