LAG COMPENSATION Reference Chap 7 OGATA

\[
\frac{E_0(s)}{E_1(s)} = \frac{KcB}{T_s + 1} \frac{T_s + 1}{BT_s + 1} = Kc \left[ \frac{S + \frac{1}{T}}{S + \frac{1}{BT}} \right]
\]

B > 1

"Bode Form"

Pole Zero Form

S-plane

Bode Magnitudes

\[
T = R_1 C_1 \quad BT = R_2 C_2 \quad B = \frac{R_2 C_2}{R_1 C_1} \quad Kc = \frac{R_4 C_1}{R_3 C_2}
\]
Root Locus Approach tolag Compensation Design

Existing System

- Good/satisfactory transient response
  - Dominant pole
- Unsatisfactory ss. Characteristic (Kv, Ka etc.)

To solve we want to:
Increase o.l. gain w/o appreciably changing transient response

Characteristics of Compensator

- Limit Angle contribution of Gc(s) to say between 0-5°
- Place pole and zero close together
- Place pole and zero near origin of S-plane
  Small change of CL poles may still result

\[ Gc(s) = K_c \frac{s + \frac{1}{\tau}}{s + \frac{1}{\beta T}} \quad B > 1 \quad \text{Let} + \frac{S}{L} \text{ Be a dominant pole} \]

\[ \left| \frac{K_c}{\frac{s + \frac{1}{\tau}}{s + \frac{1}{\beta T}}} \right| \Rightarrow |G(s)|^n = \left| \frac{K_c}{\frac{s + 1.01}{s + 0.01}} \right| = \frac{K_c}{2.89} \]
\[ \Phi G_c(s) \bigg|_{s_i} = 4K_c + 4s_i + \frac{1}{2} - 4s_i + \frac{1}{\beta T} \]

\[ -5 \leq \Phi G_c(s_i) \leq 0 \] Nominally negative angle due to lag

If \( \overset{\wedge}{K_c} \approx 1 \)

- Gain effect of lag compensator will not change open loop gain or location of dominant poles of C.L. system
- Transient response essentially unaltered

\[ \overset{\wedge}{K_c} = \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \]

- Allows us to make \( B \) large
- Close to origin
- Pole + Zero close to each other

Then gain of C.L. transfer function is increased by \( B \)

Results in an increase in static error coefficients or constants

This is due to separation of X/10
Ramp Coefficient

\[ K_v = \lim_{s \to 0} sG(s) \quad \text{UNCOMPENSATED SYSTEM} \]

\[ \hat{K}_v = \lim_{s \to 0} sG(s)G_c(s) = \lim_{s \to 0} sG(s) \frac{K_c}{s + \frac{1}{BT}} \]

\[ \therefore \hat{K}_v = K_v \lim_{s \to 0} \left[ \frac{K_c}{s + \frac{1}{BT}} \right] = K_v K_c \frac{1}{\frac{1}{BT}} = K_c K_v B \]

If \( K_c = 1 \)

Then ramp error coefficient \( K_v \) is increased by factor of \( \beta \) or mainly "controlled" by \( \beta \)

Negative effect

A closed loop pole near origin

\[ X \]

\[ \Rightarrow \text{Slow transient die out} \]

\[ \uparrow \text{Settling Time} \]

Closed loop pole between pole and zero of lag compensation
THE PROCESS TO DESIGN COMPENSATOR

**Given:**

1. Assume uncompensated system meets transient response specs. via gain adjustment. 2. Assume Ku is not correct or does not meet specs.

1) Draw RL of uncompensated system, adjust k to get dominant poles

2) Evaluate ss error coefficients of uncompensated system
   
   \[ Ku = \lim_{s \to 0} sG(s) \] (Velocity)

3) Determine amount of increase of Ku to meet specifications (multiplicative)

4) Use compensation \[ G(s)G_c(s) = G(s) \frac{K_c}{\frac{s + \frac{1}{T}}{s + \frac{1}{BT}}} \] \[ \beta > 1 \]

5) Determine pole and zero location to obtain increase in static error coefficients

Pole -3520 \( \rightarrow \) Separation is function of \( \beta \)
Ratio of $\frac{K_v \text{ spec}}{K_v \text{ system}} = \frac{\text{dist zero to origin}}{\text{dist pole to origin}}$

**Existing system no G(s)**

6) Draw RL of compensated system, adjust gain to get desired dominant poles (same as Step 1) may see slight change in G(s).

7) Adjust for Kc of compensation using magnitude criterion $K_c > 1$ generally it will be some amount less.

8) Write out resulting G(s).

9) As always, find some way to check your work.
Ex 7.2

\[ G(s) = \frac{1.06}{s(s+1)(s+2)} \]

\[ \frac{C(s)}{R(s)} = \frac{1.06}{s(s+1)(s+2) + 1.06} = \frac{1.06}{(s + 0.3307 \pm j0.5864)(s + 2.3386)} \]

**Examining Dominant Pole Location**

\[ \frac{(s + 3.307 + j1.5864)(s + 3.307 - j1.5864)}{s^2 + 0.66145 + 0.4532} \]

\[ s^2 + 2\zeta\omega_n s + \omega_n^2 \]

\[ \omega_n = 0.673 \]

\[ \zeta = 0.491 \]

\[ K_n = \lim_{s \to 0} s \frac{1.06}{s(s+1)(s+2)} = \frac{1.06}{2} = 0.53 \text{ sec}^{-1} \]
$\text{SPEC } K_v \text{ DESIGN } \approx 5 \text{ sec}^{-1}$  All Bode is Acceptable

$\zeta = 0.491 \quad \mu_w = 0.673$

$R(s) \rightarrow G_e(s) \rightarrow G(s) \rightarrow C(s)$

Configuration for Compensation Added to System

$\frac{K_v 0.65}{K_v \text{ Sys}} = \frac{5}{53} = 0.9433 \approx 10$

Choose $\beta = 10$

$G_e(s) = K_c \left[ \frac{s + \frac{1}{T}}{s + \frac{1}{10T}} \right]$

Free to choose $\beta = 0.05$

$T = 20 \Rightarrow \frac{1}{T} = 0.05$

Good starting value of $\frac{1}{10T} = 0.005$

Want $T$ Large

Do not know values of $K_c$

$G_c(s) = K_c \frac{s + 0.05}{s + 0.005}$
Examining

Dual Contribution to KL at Dominant Pole (Angle Criterion)

from Compensator $G_c(s)$

\[ G_c(s) = K_c \frac{s + 0.05}{s + 0.005} \]

\[ G_c(s) \bigg|_s = -0.3307 + j0.5864 \]

\[ G_c(s_0) = 0 + \angle-0.2807 + j1.5864 - \angle-0.3257 + j0.5864 \]

\[ 0 + 115.58 - 119.05 = -3.470 \]

\[ -3 \leq \text{Contribution} \leq 0 \text{ okay} \]

The smaller... the better. Don't go beyond -5

Loop Transfer Function

\[ G(s)G_c(s) = K_c \frac{s + 0.05}{s + 0.005} \frac{1.06}{s(s+1)(s+2)} \]

Need to find value for $K_c$

Let $K = K_c(1.06)$
\[ G(s) G_c(s) = \frac{k(s+0.05)}{s(s+0.005)(s+1)(s+2)} \]

Original System

\[ s = 0.491 \]

\[ G(s) G_c(15) \]

\[ -0.31 \pm j0.55 \]

Find dominant poles on same decay ratio line for \( G(s) G_c(s) \)

- Adjust \( k \) to get value
- Find pole

\[ \left| \frac{k(s+0.05)}{s(s+0.005)(s+1)(s+2)} \right| = 1 \]

\[ -0.31 + j0.55 \]

\[ k = \left| \frac{s(s+0.005)(s+1)(s+2)}{s+0.05} \right| = 1.0235 \]

\[ k = k_c(1.06) \Rightarrow k_c = 0.9656 \]

\[ s = -0.31 + j0.55 \]
Final design of Lag Compensator

Bode Type Form (time constant form)

\[ G_c(s) = \frac{0.9656}{s + 0.05} \]

\[ G_c(s) = 9.656 \frac{20 s + 1}{200 s + 1} \]

We get Bode form if \( s = jw \)

\[ G_c(jw) = 9.656 \frac{20 j w + 1}{200 j w + 1} \]

\[ = 9.656 \frac{w}{0.05 + j} \frac{w}{0.005 + j} \]

Final ramp error coefficient

\[ K_u = \lim_{s \to 0} \frac{1.0235(s + 0.05)}{s(s + 0.005)(s + 1)(s + 2)} = 5.1175 \text{ sec}^{-1} \]

Result obtained

Fig 2-17 shows long tail of compensated system. This is consequence of pole and zero near origin.